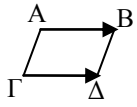
Ισότητα διανυσμάτων

$$\begin{cases} |\overrightarrow{AB}| = |\overrightarrow{\Gamma\Delta}| \\ \overrightarrow{AB} \uparrow\uparrow \overrightarrow{\Gamma\Delta} \end{cases} \Leftrightarrow \overrightarrow{AB} = \overrightarrow{\Gamma\Delta} \Leftrightarrow \begin{cases} \overrightarrow{A\Gamma} = \overrightarrow{B\Delta} \\ \overrightarrow{\Delta B} = \overrightarrow{\Gamma A} \end{cases}$$



Εσωτερικό γινόμενο διανυσμάτων

1. $\vec{\alpha}\cdot\vec{\beta} = |\vec{\alpha}||\vec{\beta}|\,\text{συν}(\vec{\alpha},\vec{\beta})$

2. $\vec{\alpha}\cdot\vec{\beta} = x_1x_2 + y_1y_2$ με $\vec{\alpha} = (x_1,y_1)$, $\vec{\beta} = (x_2,y_2)$

Απόσταση σημείου από ευθεία

Αν ε: $Ax + By + \Gamma = 0$ και $M(x_0,y_0)$, τότε:

$$d(M,\varepsilon) = \frac{|Ax_0 + By_0 + \Gamma|}{\sqrt{A^2+B^2}}$$

Απόσταση παραλλήλων

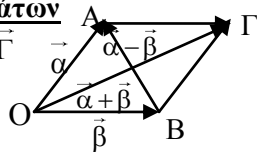
$\varepsilon_1: y = \lambda x + \beta_1$, $\varepsilon_2: y = \lambda x + \beta_2$,

$$d(\varepsilon_1,\varepsilon_2) = \frac{|\beta_1 - \beta_2|}{\sqrt{\lambda^2+1}}$$

Πρόσθεση –Αφαίρεση διανυσμάτων

$\vec{\alpha} + \vec{\beta} = \overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{A\Gamma} = \overrightarrow{O\Gamma}$

$\vec{\alpha} - \vec{\beta} = \overrightarrow{OA} - \overrightarrow{OB} = \overrightarrow{BA}$



Γωνία διανυσμάτων

$$\text{συν}(\vec{\alpha},\vec{\beta}) = \frac{\vec{\alpha}\cdot\vec{\beta}}{|\vec{\alpha}||\vec{\beta}|} = \frac{x_1x_2 + y_1y_2}{\sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2}}$$

Εμβαδόν τριγώνου

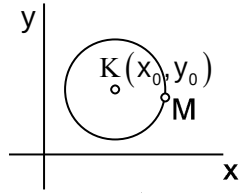
$A(x_1,y_1), B(x_2,y_2), \Gamma(x_3,y_3)$

$$E_{AB\Gamma} = \frac{1}{2}\left|\det(\overrightarrow{AB},\overrightarrow{A\Gamma})\right| = \frac{1}{2}\left|\begin{vmatrix} x_2-x_1 & y_2-y_1 \\ x_3-x_1 & y_3-y_1 \end{vmatrix}\right|$$

Κύκλος

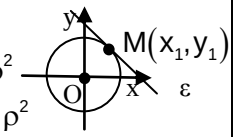
Ορισμός $(KM) = \rho$

$(x-x_0)^2 + (y-y_0)^2 = \rho^2$



$C: x^2 + y^2 = \rho^2$

$\varepsilon: xx_1 + yy_1 = \rho^2$



1η συνθήκη παραλληλίας: $\vec{\alpha} \parallel \vec{\beta} \overset{\lambda\neq 0}{\Leftrightarrow} \vec{\alpha} = \lambda\vec{\beta}$

Αν $\lambda > 0$, τότε $\vec{\alpha} \uparrow\uparrow \vec{\beta}$, αν $\lambda < 0$ τότε $\vec{\alpha} \uparrow\downarrow \vec{\beta}$

2η συνθήκη παραλληλίας

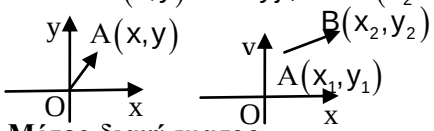
$$\vec{\alpha} \parallel \vec{\beta} \Leftrightarrow \det(\vec{\alpha},\vec{\beta}) = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = 0$$

Διχοτόμος γωνίας και Μεσοπαράλληλος: Αν

Μ σημείο της διχοτόμου ή της μεσοπαράλληλου των $\varepsilon_1, \varepsilon_2$ τότε: $d(M,\varepsilon_1) = d(M,\varepsilon_2)$

Συντεταγμένες διανύσματος

$\vec{\alpha} = \overrightarrow{OA} = (x,y) = x\vec{i} + y\vec{j}$, $\overrightarrow{AB} = (x_2-x_1, y_2-y_1)$



Μέτρο διανύσματος

$|\vec{\alpha}| = \sqrt{x^2+y^2}$, $|\overrightarrow{AB}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Συντελεστής διεύθυνσης

Αν $\vec{\alpha} = (x,y)$, τότε $\lambda_{\vec{\alpha}} = \frac{y}{x} = \varepsilon\varphi\theta$, $\theta = (x'x, \vec{\alpha})$

$\theta \in [0,\pi)$

3η συνθήκη παραλληλίας: $\vec{\alpha} \parallel \vec{\beta} \Leftrightarrow \lambda_{\vec{\alpha}} = \lambda_{\vec{\beta}}$

Παραβολή

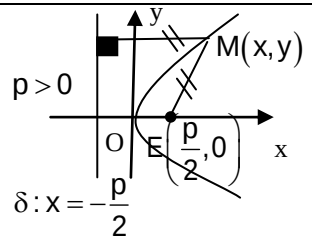
Ορισμός

$d(M,\delta) = (ME)$

$C: y^2 = 2px$

Εφαπτομένη

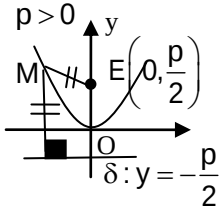
$yy_1 = p(x + x_1)$



$C: x^2 = 2py$

Εφαπτομένη

$xx_1 = p(y + y_1)$



Κάθετα διανύσματα

1. $\vec{\alpha} \perp \vec{\beta} \Leftrightarrow \vec{\alpha}\cdot\vec{\beta} = 0$ 2. $\vec{\alpha} \perp \vec{\beta} \Leftrightarrow \lambda_{\vec{\alpha}}\cdot\lambda_{\vec{\beta}} = -1$

Προβολή διανύσματος σε διάνυσμα

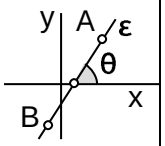
$\vec{\alpha}\cdot\vec{\beta} = \vec{\alpha}\cdot\text{προβ}_{\vec{\alpha}}\vec{\beta}$

Συντελεστής διεύθυνσης ευθείας

$$\lambda = \varepsilon\varphi\theta = \frac{y_2-y_1}{x_2-x_1}$$

$\theta \in (0^\circ, 90^\circ) \Leftrightarrow \lambda > 0$

$\theta \in (90^\circ, 180^\circ) \Leftrightarrow \lambda < 0$



Έλλειψη

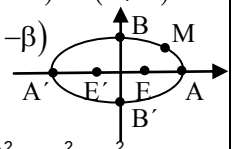
Ορισμός:

$(ME) + (ME') = 2\alpha > 0$

$A'(-\alpha,0), A(\alpha,0), E'(-\gamma,0)$,

$B(0,\beta), B'(0,-\beta)$

$E(\gamma,0)$



$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1, \beta^2 = \alpha^2 - \gamma^2$$

Συνευθειακά σημεία

Α,Β,Γ συνευθειακά $\Leftrightarrow \overrightarrow{AB} \parallel \overrightarrow{B\Gamma}$

Μέσο διανύσματος

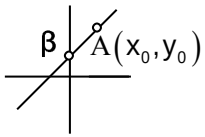
Αν $M(x_M,y_M)$, το μέσο του διανύσματος $\overrightarrow{AB}$, όπου $A(x_1,y_1)$ και $B(x_2,y_2)$, τότε:

$$\overrightarrow{OM} = \frac{\overrightarrow{OA} + \overrightarrow{OB}}{2}, x_M = \frac{x_1+x_2}{2}$$

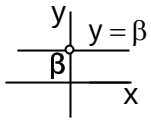
$$y_M = \frac{y_1+y_2}{2}$$

Εξίσωση ευθείας

$y - y_0 = \lambda(x - x_0)$ ή $y = \lambda x + \beta$

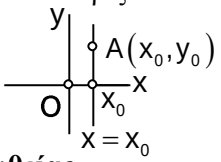


$y = \beta$, $\lambda = \varepsilon\varphi 0 = 0$



$x = x_0$, $\theta = 90^\circ$

λ= δεν ορίζεται



Γενική μορφή εξίσωσης ευθείας

$\varepsilon: Ax + By + \Gamma = 0$ με $A \neq 0$ ή $B \neq 0$, $\lambda_\varepsilon = -\frac{A}{B}$.

Διάνυσμα κάθετο και παράλληλο στην ε

$\vec{\delta} \parallel \varepsilon$ με $\vec{\delta} = (B,-A)$ και $\vec{\eta} \perp \varepsilon$ με $\vec{\eta} = (A,B)$

Υπερβολή

Ορισμός

$|(ME) - (ME')| = 2\alpha > 0$

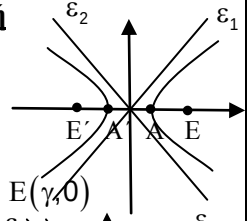
$$C: \frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1, \beta^2 = \gamma^2 - \alpha^2$$

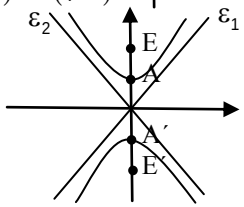
$A'(-\alpha,0), A(\alpha,0), E'(-\gamma,0), E(\gamma,0)$

$\varepsilon_1: y = \frac{\beta}{\alpha}x, \varepsilon_2: y = -\frac{\beta}{\alpha}x$

εκκεντρότητα: $\varepsilon = \frac{\gamma}{\alpha} > 1$

$$C: \frac{y^2}{\alpha^2} - \frac{x^2}{\beta^2} = 1$$

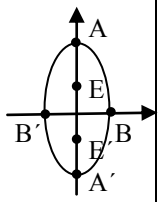




$A'(0,-\alpha), A(0,\alpha)$

$B'(-\beta,0), B(\beta,0)$,

$E'(0,-\gamma), E(0,\gamma)$.



$$C: \frac{x^2}{\beta^2} + \frac{y^2}{\alpha^2} = 1, \beta^2 = \alpha^2 - \gamma^2$$

εκκεντρότητα: $\varepsilon = \frac{\gamma}{\alpha}, 0 < \varepsilon < 1$

εφαπτομένη: $\frac{xx_1}{\alpha^2} + \frac{yy_1}{\beta^2} = 1$