

μ

μ

9

1. $\mu = 2\sqrt{3}, \quad = 1 \quad \hat{=} = 30^\circ.$
 $= \sqrt{7}.$

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2. $\mu = 2\sqrt{7}, \quad = 4 \quad = 6.$
 $\hat{=} = 60^\circ.$

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3. $\frac{\mu}{\mu} = \frac{2}{3}, \quad \frac{\mu}{\mu} = \frac{8}{9} \quad \frac{\mu}{\mu} = \frac{1}{3}.$ $\mu \quad \mu \quad \mu$
 $(\quad) = \frac{2}{3}(\quad).$

4. $\mu = \frac{1}{3}, \quad = \frac{1}{3}, \quad = \frac{1}{3}.$ $\mu \quad \mu \quad \mu$
 $(\quad) = \frac{7}{3}(\quad).$

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5. $\mu \quad \mu \quad \hat{=} > 90^\circ \quad = 5, \quad = 3, \quad (\quad) = \frac{15\sqrt{3}}{4}.$

6. $\mu \quad \mu \quad // \quad < \quad \mu \quad \mu$
 $(\quad) = (\quad) \quad (\quad) + (\quad) = (\quad) \quad (\quad) - (\quad) = (\quad)$

7. $\mu \quad \mu \quad \mu \quad = \frac{2}{3} \quad = \frac{3}{4}.$
 $(\quad) = (\quad) \quad (\quad) = (\quad)$

8. $\mu = 14, \quad = 10, \quad = 6$
 $\mu \quad \mu \quad \mu \quad 15\sqrt{3}.$

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9. $\mu = 4, \quad = 2\sqrt{3} \quad \hat{=} = 60^\circ.$
 $\mu \quad \mu$
 $\mu \quad \mu \quad = \frac{1}{3}, \quad \frac{(\quad)}{(\quad)} = \frac{1}{6}.$

10. $\mu = \frac{1}{4}$, $\mu = \frac{1}{8}$, $\mu = \frac{3}{8}$:

$(\frac{1}{4}) = (\frac{1}{8}) = \frac{1}{4}(\frac{1}{8})$

$(\frac{1}{8}) = \frac{1}{8}(\frac{1}{8})$

$(\frac{3}{8}) = \frac{3}{8}(\frac{1}{8})$

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11. $\mu = 2$, $\mu = \frac{1}{3}$:

$(\frac{1}{3}) = (\frac{1}{3})$

$(\frac{1}{2}) = \frac{1}{2}(\frac{1}{2})$

$(\frac{1}{3}) = \frac{1}{3}(\frac{1}{3})$

$(\frac{2}{3}) = \frac{2}{3}(\frac{1}{3})$

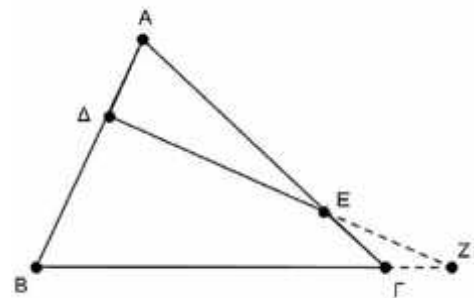
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12. $\mu = \frac{1}{3}$, $\mu = \frac{1}{4}$:

$(\frac{1}{4}) = \frac{1}{4}(\frac{1}{4})$

$(\frac{1}{4}) = \frac{1}{4}(\frac{1}{4})$, $(\frac{2}{3}) = \frac{2}{3}(\frac{1}{3})$

$(\frac{1}{5}) = \frac{1}{5}(\frac{1}{5})$



13. $\mu = 90^\circ$, $\mu = 6$, $\mu = 8$:

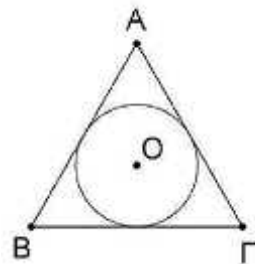
$(\frac{1}{6}) = \frac{1}{6}(\frac{1}{6})$, $(\frac{1}{8}) = \frac{1}{8}(\frac{1}{8})$

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14. $\mu = 9$, $\mu = 6\sqrt{3}$:

$(\frac{1}{9}) = \frac{1}{9}(\frac{1}{9})$, $(\frac{1}{6\sqrt{3}}) = \frac{1}{6\sqrt{3}}(\frac{1}{6\sqrt{3}})$

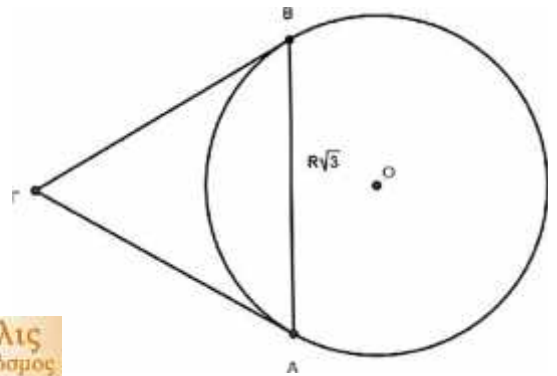
$(\frac{1}{9}) = \frac{1}{9}(\frac{1}{9})$, $(\frac{1}{6\sqrt{3}}) = \frac{1}{6\sqrt{3}}(\frac{1}{6\sqrt{3}})$



22. $(, R)$

$\mu R\sqrt{3}$

$\mu \mu \mu$



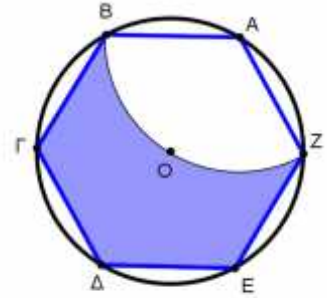
$R^2 \left(\sqrt{3} - \frac{1}{3} \right)$

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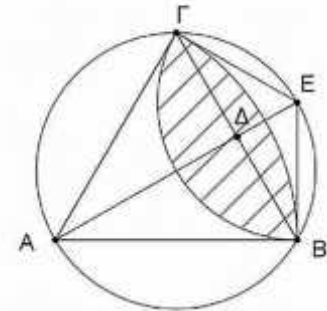
23. (O, R)

$R \mu \mu$

$B\hat{A}Z$



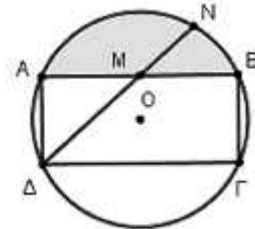
$\mu \mu \mu = \frac{\sqrt{3}}{6}$



25. $(, R) = R$

$\mu \mu \mu$

$\mu \mu \mu$



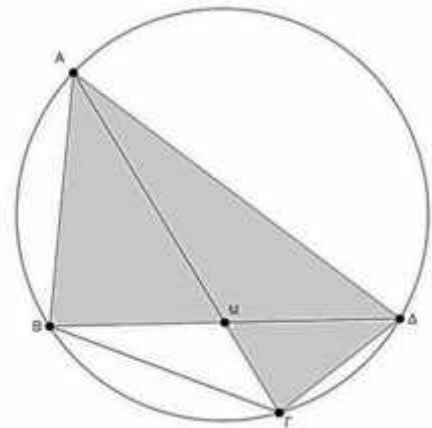
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26. $\mu \mu \mu AM = \frac{4}{5}$

$() = 4()$

i. $() = \frac{2}{32} \mu$

ii. $() = \frac{9}{32} \mu$



27. μ μ $\mu\mu$ (μ , R)

μ μ $\mu\mu$

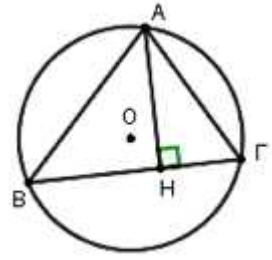
) $\hat{\mu} = 75^\circ$.

) $= \frac{R\sqrt{6}}{2}$.

) R.

) μ μ $\mu\mu$

μ , μ , R.



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28. μ μ μ μ = 8,

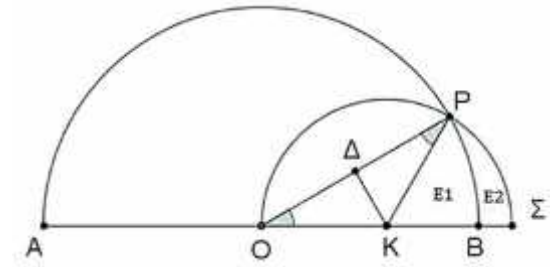
$= \frac{8\sqrt{3}}{3}$.

) $\hat{\mu} = 120^\circ$.

) μ $\frac{1}{3}$ μ $\mu\mu$

) μ $\frac{4}{3}(-\sqrt{3})$.

) μ 2 μ $\mu\mu$



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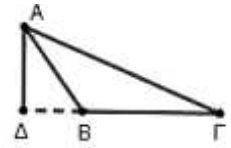
1. $\mu = 2\sqrt{3}, \alpha = 1, \hat{\alpha} = 30^\circ$
 $\mu = \sqrt{7}$
 μ

$\mu^2 = \mu^2 + \mu^2 - 2 \mu \mu \cos \hat{\alpha}$
 $= 1 + 12 - 4\sqrt{3} \frac{\sqrt{3}}{2} = 7 \Leftrightarrow \mu = \sqrt{7}$

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$\mu^2 = 12, \mu^2 + \mu^2 = 8, \mu^2 > \mu^2 + \mu^2 \hat{\alpha} > 90^\circ$

$\hat{\alpha} > 90^\circ, \mu^2 = \mu^2 + \mu^2 + 2 \mu \mu \cos \hat{\alpha} \Leftrightarrow 12 = 8 + 2 \mu^2 \Leftrightarrow \mu = 2$



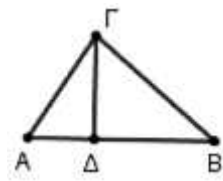
2. $\mu = 2\sqrt{7}, \alpha = 4, \beta = 6$
 $\hat{\alpha} = 60^\circ$

$\mu^2 + \mu^2 = 28 + 16 = 44 > 36 = \alpha^2 \hat{\alpha} < 90^\circ$

$\mu^2 = \mu^2 + \mu^2 - 2 \mu \mu \cos \hat{\alpha} \Leftrightarrow 28 = 16 + 36 - 48 \cos \hat{\alpha} \Leftrightarrow 48 \cos \hat{\alpha} = 24 \Leftrightarrow \cos \hat{\alpha} = \frac{1}{2} \Leftrightarrow \hat{\alpha} = 60^\circ$

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$\mu^2 = \mu^2 + \mu^2 - 2 \mu \mu \cos \hat{\alpha} \Leftrightarrow 28 = 16 + 36 - 12 \cos \hat{\alpha} \Leftrightarrow \cos \hat{\alpha} = \frac{1}{2} \Leftrightarrow \hat{\alpha} = 60^\circ$

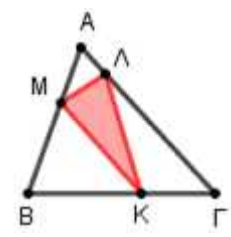


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3. $\frac{1}{3} = \frac{2}{3}, \frac{1}{9} = \frac{8}{9}, \frac{1}{3} = \frac{1}{3}, (\frac{1}{3}) = \frac{2}{3} (\frac{1}{3})$

$\frac{(\frac{1}{3})}{(\frac{1}{3})} = \frac{1}{3} \cdot \frac{1}{9} = \frac{1}{27} \Leftrightarrow (\frac{1}{3}) = \frac{1}{27} (\frac{1}{3})$ (1)

$\frac{(\frac{2}{3})}{(\frac{2}{3})} = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9} \Leftrightarrow (\frac{2}{3}) = \frac{4}{9} (\frac{2}{3})$



$$\frac{(\quad)}{(\quad)} = \frac{\cdot}{\cdot} = \frac{1 \cdot 8}{3 \cdot 9} = \frac{8}{27} \Leftrightarrow (\quad) = \frac{8}{27} (\quad)$$

$$(\quad) = (\quad) - (\quad) - (\quad) - (\quad) \stackrel{(1),(2),(3)}{=} (\quad) - \frac{1}{27} (\quad) - \frac{4}{9} (\quad) - \frac{8}{27} (\quad) = \frac{2}{3} (\quad)$$

4.

$$\mu = \frac{1}{3}, \quad \mu = \frac{1}{3}, \quad \mu = \frac{1}{3}$$

$$(\quad) = \frac{7}{3} (\quad)$$

$$\hat{\quad} + \hat{\quad} = 180^\circ$$

$$\frac{(\quad)}{(\quad)} = \frac{\frac{1}{3} \cdot 4}{\frac{1}{3} \cdot 3} = \frac{4}{9} \Leftrightarrow (\quad) = \frac{4}{9} (\quad)$$

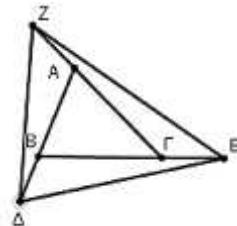
$$\hat{\quad} + \hat{\quad} = 180^\circ$$

$$\frac{(\quad)}{(\quad)} = \frac{\frac{1}{3} \cdot 4}{\frac{1}{3} \cdot 3} = \frac{4}{9} \Leftrightarrow (\quad) = \frac{4}{9} (\quad)$$

$$\hat{\quad} + \hat{\quad} = 180^\circ$$

$$\frac{(\quad)}{(\quad)} = \frac{\frac{4}{3} \cdot \frac{1}{3}}{\frac{4}{3} \cdot 3} = \frac{4}{9} \Leftrightarrow (\quad) = \frac{4}{9} (\quad)$$

$$(\quad) = (\quad) + (\quad) + (\quad) + (\quad) = (\quad) + \cancel{\frac{4}{9}} (\quad) = \frac{7}{3} (\quad)$$



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5.

$$\mu > 90^\circ = 5, \quad \mu = 3, \quad (\quad) = \frac{15\sqrt{3}}{4}$$

$$(\quad) = \frac{1}{2} \cdot \mu \Leftrightarrow \frac{15\sqrt{3}}{4} = \frac{1}{2} \cdot 5 \cdot 3 \mu \Leftrightarrow \mu = \frac{\sqrt{3}}{2} = \mu 60^\circ$$

$$\hat{\quad} > 90^\circ, \quad \hat{\quad} = 180^\circ - 60^\circ = 120^\circ$$

$$^2 = ^2 + ^2 - 2 \cdot \cdot \cdot 120^\circ = 25 + 9 - 30 \left(-\frac{1}{2}\right) = 49 \Leftrightarrow = 7$$

$$(\quad) = \frac{\quad}{4R} \Leftrightarrow 4R(AB) = \quad \Leftrightarrow R = \frac{7 \cdot \cancel{5} \cdot \cancel{3}}{\cancel{15} \sqrt{3}} = \frac{7\sqrt{3}}{3}$$

$$\mu \quad \mu \quad \mu \quad : = R^2 = \left(\frac{7}{\sqrt{3}}\right)^2 = \frac{49}{3}$$

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6. μ // $<$, μ ,
 μ
 $() () = () () + () = () () - () = ()$

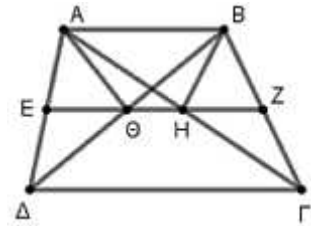
) μ // // $= \frac{+}{2}$.

$() = () = \frac{1}{2} ()$ (1)

$() = () = \frac{1}{2} ()$ (2)

μ $() = ()$ (3)

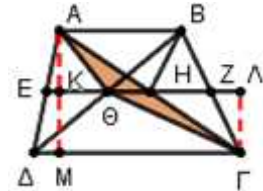
(1),(2), (3) $() = ()$.



) $() + () = \frac{1}{2} () + \frac{1}{2} () = \frac{1}{2} () + \frac{1}{2} () = ()$

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)
 $() - () = \frac{(+) K}{2} - \frac{(+) K}{2} =$
 $\frac{(+) A}{4} - \frac{(+) K}{4} =$
 $= \frac{(+ / - - /)}{4} = \frac{(-)}{4}$ (4)



$() = () + () = \frac{1}{2} \cdot + \frac{1}{2} \cdot = \frac{1}{2} \cdot + \frac{1}{2} \cdot =$
 $= \frac{1}{2} \cdot (+) = \frac{1}{2} \cdot$

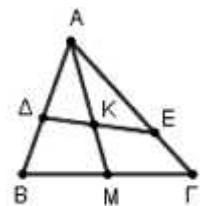
μ μ μ μ $= \frac{-}{2}$

$() = \frac{1}{2} \cdot \frac{-}{2} \Leftrightarrow () = \frac{(-)}{4}$ (5)

(4),(5) $() - () = ()$.

7. μ μ μ $= \frac{2}{3}$ $= \frac{3}{4}$.
 μ μ
 $() () = () () = ()$

) , :
 $\frac{(A)}{()} = \frac{\frac{2}{3} \cdot \frac{3}{4}}{\frac{3}{4}} = \frac{1}{2} \Leftrightarrow (A) = \frac{1}{2} ()$,



() = $\frac{1}{2}$ (), () = ()



) $\frac{()}{()} = \frac{\cdot /}{\cdot /} = \frac{1}{2} \Leftrightarrow () = \frac{1}{2}()$, () = () .

: () = () $\Leftrightarrow () + (/) = () + (/) \Leftrightarrow () = ()$

8. $\mu = 14, = 10, = 6$
) μ
) μ $15\sqrt{3}$.

) $^2 = 144 > ^2 + ^2 = 136 \Leftrightarrow \hat{ } > 90^\circ$.



) $= \frac{+ +}{2} = 15$

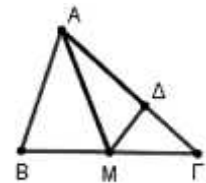
() = $\sqrt{(-)(-)(-)} = \sqrt{15 \cdot 1 \cdot 5 \cdot 9} = 3\sqrt{75} = 15\sqrt{3}$

9. $\mu = 4, = 2\sqrt{3} \hat{ } = 60^\circ$.

) μ μ $= \frac{1}{3}$, $\frac{()}{()} = \frac{1}{6}$.

) () = $\frac{1}{2} \cdot \cdot \mu = \frac{1}{2} \cdot 4 \cdot 2\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 6 \mu$.

) $\frac{()}{()} = \frac{\cdot /}{\cdot /} = \frac{\frac{1}{2} /}{\frac{1}{3} /} = \frac{1}{6}$.



) () = () = $\frac{1}{2}$ () .

$\frac{()}{()} = \frac{\cdot /}{\cdot /} = \frac{\frac{2}{3} /}{/} = \frac{2}{3} \Leftrightarrow () = \frac{2}{3}() = \frac{2}{3} \cdot \frac{1}{2}() = \frac{1}{3} \cdot 6 = 2 \mu$.

$\frac{()}{()} = \frac{1}{6} \Leftrightarrow \frac{()}{6} = \frac{1}{6} \Leftrightarrow () = 1$

(A) = () - () = $\frac{()}{2} - () = 3 - 1 = 2$.

10.

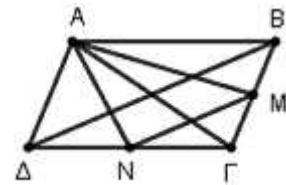
$$\left(\frac{\mu}{\mu}\right) = \left(\frac{\mu}{\mu}\right) = \frac{1}{4} \left(\frac{\mu}{\mu}\right)$$

$$\left(\frac{\mu}{\mu}\right) = \frac{1}{8} \left(\frac{\mu}{\mu}\right)$$

$$\left(\frac{\mu}{\mu}\right) = \frac{3}{8} \left(\frac{\mu}{\mu}\right)$$

$$\left(\frac{\mu}{\mu}\right) = \left(\frac{\mu}{\mu}\right) = \frac{1}{2} \left(\frac{\mu}{\mu}\right) = \frac{1}{2} \cdot \frac{1}{2} \left(\frac{\mu}{\mu}\right) = \frac{1}{4} \left(\frac{\mu}{\mu}\right)$$

$$\left(\frac{\mu}{\mu}\right) = \left(\frac{\mu}{\mu}\right) = \frac{1}{2} \left(\frac{\mu}{\mu}\right) = \frac{1}{2} \cdot \frac{1}{2} \left(\frac{\mu}{\mu}\right) = \frac{1}{4} \left(\frac{\mu}{\mu}\right)$$



) :

$$\left(\frac{\mu}{\mu}\right) = \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{2}} = \frac{1}{4} \Leftrightarrow \left(\frac{\mu}{\mu}\right) = \frac{1}{4} \left(\frac{\mu}{\mu}\right) = \frac{1}{4} \cdot \frac{1}{2} \left(\frac{\mu}{\mu}\right) = \frac{1}{8} \left(\frac{\mu}{\mu}\right)$$

$$\left(\frac{\mu}{\mu}\right) = \left(\frac{\mu}{\mu}\right) - \left(\frac{\mu}{\mu}\right) = \left(\frac{\mu}{\mu}\right) + \left(\frac{\mu}{\mu}\right) - \left(\frac{\mu}{\mu}\right) \Leftrightarrow$$

$$\left(\frac{\mu}{\mu}\right) = \frac{1}{4} \left(\frac{\mu}{\mu}\right) + \frac{1}{4} \left(\frac{\mu}{\mu}\right) - \frac{1}{8} \left(\frac{\mu}{\mu}\right) = \frac{3}{8} \left(\frac{\mu}{\mu}\right)$$

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$$\mu = 2$$

$$\mu = \frac{1}{3}, \mu = \frac{1}{3}$$

$$\left(\frac{\mu}{\mu}\right) = \left(\frac{\mu}{\mu}\right)$$

$$\left(\frac{\mu}{\mu}\right) = \frac{1}{2} \left(\frac{\mu}{\mu}\right)$$

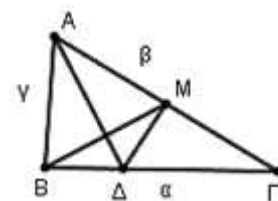
$$\left(\frac{\mu}{\mu}\right) = \frac{1}{3} \left(\frac{\mu}{\mu}\right)$$

$$\left(\frac{\mu}{\mu}\right) = \frac{2}{3} \left(\frac{\mu}{\mu}\right)$$

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) : $\hat{\alpha} = \hat{\beta}$

$$\left(\frac{\mu}{\mu}\right) = \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{2}} = 1 \Leftrightarrow \left(\frac{\mu}{\mu}\right) = \left(\frac{\mu}{\mu}\right)$$



) : $\hat{\alpha} + \hat{\beta} = 180^\circ$

$$\left(\frac{\mu}{\mu}\right) = \frac{\frac{1}{\beta}}{\frac{2}{\beta}} = \frac{1}{2}$$

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) A , :

$$\frac{(\quad)}{(\quad)} = \frac{\cdot}{\cdot} = \frac{2 \cdot \cancel{3}}{\cancel{3} \cdot \cdot} = \frac{2}{3} \Leftrightarrow (\quad) = \frac{2}{3} (\quad) \Leftrightarrow \cancel{3} (\quad) = \frac{\cancel{3}}{3} (\quad) \Leftrightarrow$$

$$(\quad) = \frac{1}{3} (\quad)$$

) $(\quad) = (\quad) - (\quad) = (\quad) - \frac{1}{3} (\quad) = \frac{2}{3} (\quad)$

12.

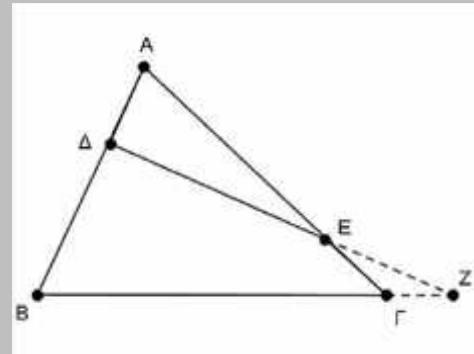
μ , : $\frac{1}{3} = \frac{1}{4}$.

μ .

) : $\frac{(\quad)}{(\quad)}$.

) : $\frac{(\quad)}{(\quad)} = \frac{1}{4} \cdot \text{---}$ $\frac{(\quad)}{(\quad)} = \frac{2}{3} \cdot \text{---}$.

) $= \frac{1}{5}$.



) , :

$$\frac{(\quad)}{(\quad)} = \frac{\cdot}{\cdot} = \frac{1 \cdot \cancel{3} \cdot \cancel{4}}{\cancel{3} \cdot \cancel{4} \cdot \cdot} = \frac{1}{4} \Leftrightarrow (\quad) = \frac{1}{4} (\quad) .$$

$$\frac{(\quad)}{(\quad)} = \frac{(\quad) - (\quad)}{(\quad)} = 1 - \frac{(\quad)}{(\quad)} = 1 - \frac{1}{4} = \frac{3}{4}$$

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) $\hat{\quad} + \hat{\quad} = 180^\circ$:

$$\frac{(\quad)}{(\quad)} = \frac{\cdot}{\cdot} = \frac{1 \cdot \cancel{4}}{\cancel{4} \cdot \cdot} = \frac{1}{4}$$

, :

$$\frac{(\quad)}{(\quad)} = \frac{\cdot}{\cdot} = \frac{2 \cdot \cancel{3}}{\cancel{3} \cdot \cdot} = \frac{2}{3}$$

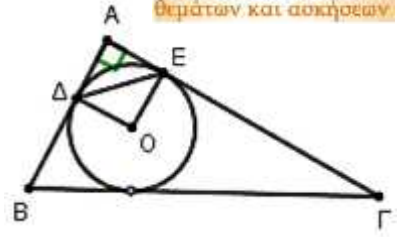
) $\frac{(\quad)}{(\quad)} = \frac{3}{4} \Leftrightarrow \frac{(\quad) - (\quad)}{(\quad)} = \frac{3}{4} \Leftrightarrow \frac{(\quad)}{(\quad)} - \frac{(\quad)}{(\quad)} = \frac{3}{4} \Leftrightarrow \frac{2}{3} \cdot \text{---} - \frac{1}{4} \cdot \text{---} = \frac{3}{4} \Leftrightarrow$

$$8 \cdot \text{---} - 3 \cdot \text{---} = 9 \Leftrightarrow 8(\quad + \quad) - 3 \cdot \text{---} = 9 \Leftrightarrow 8 \cdot \text{---} + 8 \cdot \text{---} - 3 \cdot \text{---} = 9 \Leftrightarrow$$

$$5 \cdot \text{---} = 9 \Leftrightarrow \text{---} = \frac{9}{5}$$

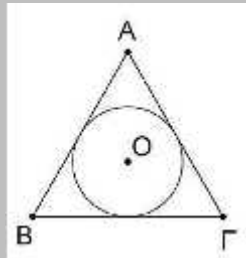
13. $\mu \hat{=} 90^\circ, \mu = 6 = 8$
 μ , $(,)$, μ ,
 $)$ μ
 $)$ μ

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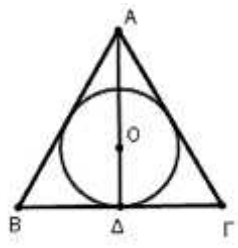


$)$ $\mu \mu$:
 $2 = 6^2 + 8^2 = 100 \Leftrightarrow \mu = 10$
 $= \frac{6^2 + 8^2}{2} = 12 \quad () = \frac{1}{2} \cdot = 24$
 $\mu () = \Leftrightarrow 24 = 12 \Leftrightarrow = 2$
 $)$ \perp , \perp $OA = OE$
 $\cdot () = \frac{1}{2} \cdot = \frac{1}{2} \cdot 2 = 2 \mu$

14. $\mu \mu$,
 $(,)$ μ 9μ
 $)$ $\mu \mu$
 $)$ $= 6\sqrt{3}$
 $)$ $\mu \mu$
 $)$ μ



$)$ $2 = 9 \Leftrightarrow 2 = 9 \Leftrightarrow = 3$
 $)$ $\mu \mu \mu$, μ ,
 μ :
 $2 = 2 - \left(\frac{2}{2}\right)^2 = \frac{3^2}{4} \Leftrightarrow = \frac{\sqrt{3}}{2}$
 $= \frac{1}{3} = \frac{1}{3} \cdot \frac{\sqrt{3}}{2} \Leftrightarrow = \frac{6}{\sqrt{3}} = 2\sqrt{3} = 6\sqrt{3}$



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2
 $(AB\Gamma) = \tau\rho \Leftrightarrow \frac{\alpha^2 \sqrt{3}}{4} = \frac{3}{2} \cdot 3 \Leftrightarrow \alpha = \frac{18}{\sqrt{3}} = 6\sqrt{3}$

$)$ $R \mu \mu$: $R = = = \frac{2}{3} = \frac{2}{3} \cdot \frac{\sqrt{3}}{2} = \frac{6(\sqrt{3})^2}{3} = 6$

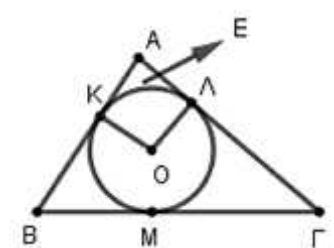
2
 $(AB\Gamma) = \frac{\alpha^3}{4R} \Leftrightarrow \frac{\alpha^2 \sqrt{3}}{4} = \frac{\alpha^3}{4R} \Leftrightarrow R = \frac{\alpha}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{3}} = 6$

$)$ $= = 3 \cdot 3 = 9 \cdot 6\sqrt{3} = 54\sqrt{3}$

11

15. (,) μμ (, R) μμ
) , R : μμ μμ
) μ μ μμ μ μ μ μμ
 (,), μμ

) = , = μ
 $2^2 = 2^2 + 2^2 = 2 \cdot 2^2 \Leftrightarrow = \sqrt{2}$
 $\mu () = \frac{\sqrt{2} \cdot \cdot}{4R} = \frac{3\sqrt{2}}{4R}$
 $() = (\sqrt{2} + +) = (2 + \sqrt{2})$
 $\frac{\beta^2 \sqrt{2}}{4R} = \frac{1}{(2 + \sqrt{2})} \Leftrightarrow 4R (2 + \sqrt{2}) = 2\sqrt{2} \Leftrightarrow R = \frac{2\sqrt{2}}{4(2 + \sqrt{2})}$



μ $\widehat{K} = \widehat{\Lambda} = 90^\circ$, $= 4 = R\sqrt{2}$,
 $R = \frac{\cancel{4} R^2 \sqrt{2}}{4^2 (2 + \sqrt{2})} \Leftrightarrow 2 (2 + \sqrt{2}) = R\sqrt{2} \Leftrightarrow R = \frac{2 (2 + \sqrt{2})}{\sqrt{2}} = \frac{2\sqrt{2} (2 + \sqrt{2})}{(\sqrt{2})^2} = \frac{\cancel{2} \sqrt{2} (2 + \sqrt{2})}{\cancel{2}}$



2
 $2^2 = 2^2 + 2^2 = 2 \cdot 2^2 \Leftrightarrow = \sqrt{2}$
 $\mu \mu \mu \mu \mu$
 $R = \frac{B\Gamma}{2} = \frac{\beta\sqrt{2}}{2} \Leftrightarrow \beta = R\sqrt{2}$
 $(AB\Gamma) = \frac{AB \cdot A\Gamma}{2} \Leftrightarrow \tau\rho = \frac{\beta^2}{2} \Leftrightarrow (2\beta' + \beta'\sqrt{2}) \cdot \rho = \frac{\beta^2}{2} \Leftrightarrow (2 + \sqrt{2}) \cdot \rho = \frac{R\sqrt{2}}{2} \Leftrightarrow R = \sqrt{2} (2 + \sqrt{2})$

) $OK \perp AB, O \perp \widehat{K} = 90^\circ$
 $= () - (, \widehat{K}) = 2 - \frac{2 \cdot 90^\circ}{360^\circ} = \frac{4 - 2}{4} = \frac{2}{4}$

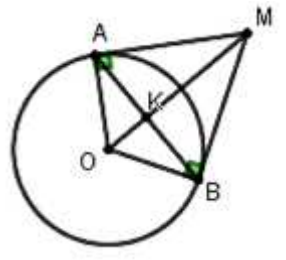
16. (, R₁), (, R₂), (, R₃) μ , ,
 $R_1 = R_2 = \sqrt{2}, R_3 = 2 - \sqrt{2}$
)
) μ μ μμ
) μ μ μμ

$$s_1 = \left(\frac{1}{2} \right) - \left(\frac{1}{2} \right) = \frac{\left(\frac{1}{2} \right)^{90^\circ}}{360^\circ} - \frac{1}{2} = \frac{1}{2} - \frac{1}{2} = \frac{1}{16} - \frac{1}{8} = \frac{1-2}{16}$$

$$s_2 = \left(\frac{1}{2} \right) - \left(\frac{1}{2} \right) = \frac{1}{16}, \quad s_1 = 2 \frac{1}{16} = \frac{1}{8}$$

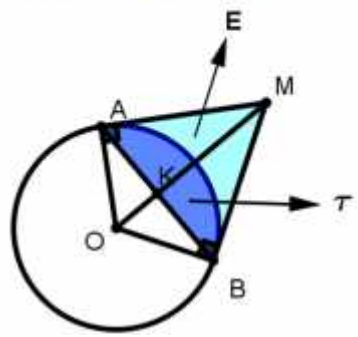
18. (, R) μ , μ μ μ
 () = 3() , :
) = 2R = R√3 .
) R, μ μ μ μ μ

) = , = , μ .
 () = 3() ⇔ $\frac{1}{2} / \cdot = 3 \cdot \frac{1}{2} / \cdot \Leftrightarrow = 3$.
 μ ⊥ .
 $2^2 = \dots = 3^2$
 $2^2 + 2^2 = 2^2 \Leftrightarrow 3^2 + O^2 = R^2 \Leftrightarrow 4^2 = R^2 \Leftrightarrow = \frac{R}{2}$. μ $s_3 = \frac{R}{2}$,
 $= s_3 = R\sqrt{3}$. MO = 4OK = 4 $\frac{R}{2}$ = 2R



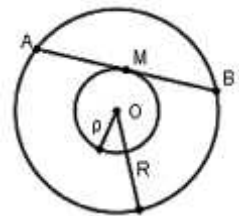
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) μ μ μ μ
 : E = (MAB) - () = $\frac{1}{2} \cdot \left(\left(\frac{1}{2} \right) - \left(\frac{1}{2} \right) \right) \Leftrightarrow$
 $E = \frac{1}{2} R \sqrt{3} \cdot 3 \frac{R}{2} - \frac{R^2 \cdot 120^\circ}{360^\circ} + \frac{1}{2} R \cdot R \cdot \mu 120^\circ = \frac{3\sqrt{3}R^2}{4} - \frac{R^2}{3} + \frac{\sqrt{3}R^2}{4} \Leftrightarrow$
 $E = \sqrt{3}R^2 - \frac{R^2}{3} = R^2 \left(\sqrt{3} - \frac{1}{3} \right)$



19. μ μ (, R) (,) μ < R μ
 μ μ μ .
) = $\frac{R\sqrt{2}}{2}$.
) μ μ μ μ , μ μ 4
 μ μ (, R) .
) μ μ μ , μ R^2 .

$$r_2 = r_1 - r_2 \Leftrightarrow \frac{r_1}{2} = \frac{r_2}{2} \Leftrightarrow \frac{R}{2} = \frac{R\sqrt{2}}{2} \Leftrightarrow \sqrt{2} = R \Leftrightarrow R = \frac{R\sqrt{2}}{2}$$



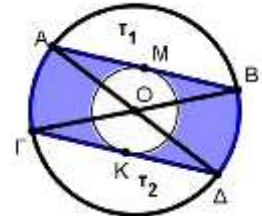
$$OM = \frac{R\sqrt{2}}{2} = r, \quad r = \frac{R\sqrt{2}}{2}$$

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$$E = R^2 - \left[(O\widehat{AB}) - (OAB) \right] - \left[(O\widehat{\Gamma}) - (O\Gamma) \right] - \left(\frac{R\sqrt{2}}{2} \right)^2 \Leftrightarrow$$

$$E = R^2 - (O\widehat{AB}) + (OAB) - (O\widehat{\Gamma}) + (O\Gamma) - \frac{R^2}{2} \Leftrightarrow$$

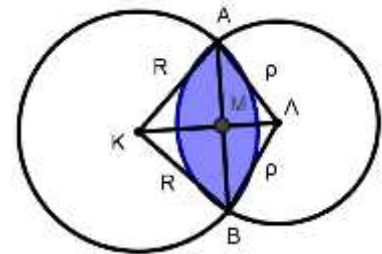
$$E = \frac{R^2}{2} - 2 \frac{R^2 \cdot 90^\circ}{360^\circ} + 2 \frac{1}{2} \cdot R \cdot R = R^2$$



20. $\mu = \dots = \dots$
 \dots
 \dots
 \dots

$$= R\sqrt{2} \Leftrightarrow R = \frac{\sqrt{2}}{2}$$

$$= \sqrt{3} \Leftrightarrow R = \frac{\sqrt{3}}{3}$$



$$M = \frac{R\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2}$$

$$M = \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{6} = \frac{3 + \sqrt{3}}{6}$$

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$$= \left(\widehat{\Gamma} \right) - \left(\Gamma \right) + \left(\widehat{\Delta} \right) - \left(\Delta \right) = \frac{R^2 \cdot 90^\circ}{360^\circ} - \frac{1}{2} R^2 + \frac{R^2 \cdot 120^\circ}{360^\circ} - \frac{1}{2} R^2 \quad \mu 120^\circ \Leftrightarrow$$

$$= \frac{R^2}{4} - \frac{1}{2} R^2 + \frac{R^2}{3} - \frac{1}{2} R^2 = \frac{-2}{4} R^2 + \frac{4 + 3\sqrt{3}}{12} R^2 \Leftrightarrow$$

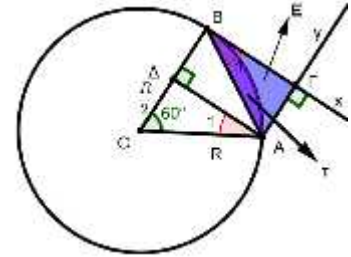
$$= \frac{-2}{4} \left(\frac{\sqrt{2}}{2} \right)^2 + \frac{4 + 3\sqrt{3}}{12} \left(\frac{\sqrt{3}}{3} \right)^2 = \frac{(-2)^2}{8} + \frac{(4 + 3\sqrt{3})^2}{36}$$

21. (O, R) μ , μ $\hat{=} 60^\circ$. μ μ x
 $y \perp Bx$ μ x . $A \perp OB$, μ μ x
)
) μ μ μ $\mu\mu$ R.
) μ μ $\mu\mu$ R.

) $OA = OB = R$ $\hat{=} 60^\circ$,
 R. $\hat{=} 60^\circ$ $\hat{A}_1 = 30^\circ$

$$O = \frac{R}{2} = \frac{R}{2}.$$

$$\mu : A^2 = R^2 - \frac{R^2}{4} = \frac{3R^2}{4} \Leftrightarrow A = \frac{R\sqrt{3}}{2}.$$



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) μ $l_{AB} = \frac{R \cdot 60^\circ}{180^\circ \cdot 3} = \frac{R}{3}$.

$$\hat{B}_1 = \frac{\hat{}}{2} = 30^\circ$$

μ

$$A = \frac{AB}{2} = \frac{R}{2}.$$

$$B = \frac{R\sqrt{3}}{2}.$$

$$= l_{AB} + \frac{R}{3} + \frac{R}{2} + \frac{R\sqrt{3}}{2} = \frac{R(2 + 3 + 3\sqrt{3})}{6}$$

2

$$\mu \quad x \quad \mu \quad \perp \quad l_{AB} = \frac{R \cdot 60^\circ}{180^\circ \cdot 3} = \frac{R}{3}$$

$$B\Gamma = A\Delta = \frac{R\sqrt{3}}{2}$$

$$A\Gamma = B\Delta = \frac{R}{2} (\mu)$$

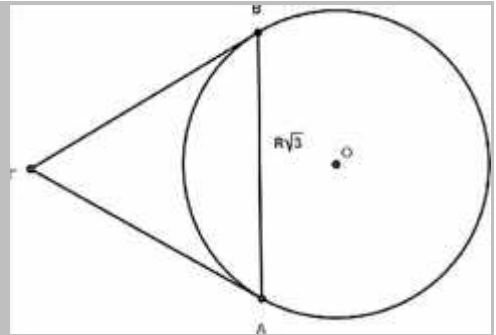
$$= l_{AB} + \frac{R}{3} + \frac{R}{2} + \frac{R\sqrt{3}}{2} = \frac{R(2 + 3 + 3\sqrt{3})}{6}$$

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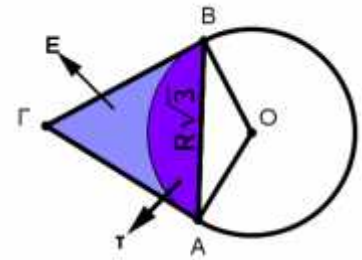
) μ μ . :
 $= (\dots) - (\dots) = \frac{1}{2} \cdot \dots - [(\dots) - (\dots)] = \frac{1}{2} \cdot \frac{R}{2} \cdot \frac{R\sqrt{3}}{2} - \frac{R^2 \cdot 60^\circ}{360^\circ \cdot 6} + \frac{1}{2} \cdot \dots \cdot \mu 60^\circ \Leftrightarrow$

$$= \frac{R^2\sqrt{3}}{8} - \frac{R^2}{6} + \frac{R^2\sqrt{3}}{4} = \frac{R^2(9\sqrt{3} - 4)}{24}$$

22. μ $R\sqrt{3}$ μ $(,R)$
 μ μ
 μ μ $\mu\mu$
 μ μ $\mu\mu$
 $R^2\left(\sqrt{3}-\frac{1}{3}\right)$



) μ μ $= R\sqrt{3} = \frac{1}{3} \mu$ $\widehat{AOB} = 120^\circ$,
 $\widehat{AOB} = \widehat{AOB} = 60^\circ$

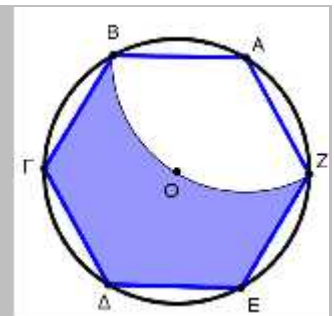


) $l_{\widehat{AB}} = \frac{R \cdot 120^\circ}{180^\circ} = \frac{2}{3} R$, μ μ :
 $= \frac{1}{2} \mu + \frac{1}{2} \mu + l_{\widehat{AB}} = 2R\sqrt{3} + \frac{2}{3} R = \frac{R(6\sqrt{3}+2)}{3}$

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) μ μ , : $= (\widehat{AOB}) - (\widehat{AOB}) = \frac{1}{2} \cdot \mu 60^\circ - \left[\left(\frac{1}{2} \mu \right) - \left(\frac{1}{2} \mu \right) \right] \Leftrightarrow$
 $= \frac{1}{2} \cdot R\sqrt{3} \cdot R\sqrt{3} \cdot \frac{\sqrt{3}}{2} - \frac{R^2 \cdot 120^\circ}{360^\circ} + \frac{1}{2} \cdot \mu 120^\circ = \frac{3R^2\sqrt{3}}{4} - \frac{R^2}{3} + \frac{R^2\sqrt{3}}{4} = \frac{R^2(3\sqrt{3}-1)}{3}$

23. $\mu\mu$ (O,R) .
 R μ μ
 μ μ $\mu\mu$
 \widehat{BAZ} .
 μ μ $\mu\mu$



) $\widehat{AOB} = \widehat{AOB} = 180^\circ - \frac{360^\circ}{6} = 120^\circ$

) $\mu = \frac{1}{2} \mu$ $\mu = \frac{1}{2} \cdot 6 \mu = 3R \frac{R\sqrt{3}}{2} = \frac{3R^2\sqrt{3}}{2}$

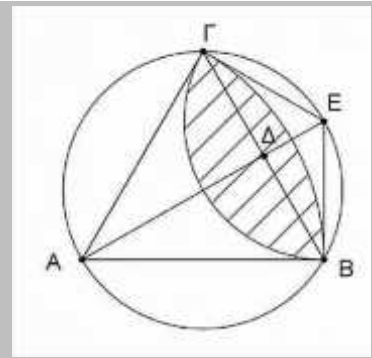
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) μ μ , : $= E_6 - (\widehat{A.B}) = \frac{3R^2\sqrt{3}}{2} - \frac{R^2 \cdot 120^\circ}{360^\circ} = \frac{(9\sqrt{3}-2)R^2}{6}$

24.

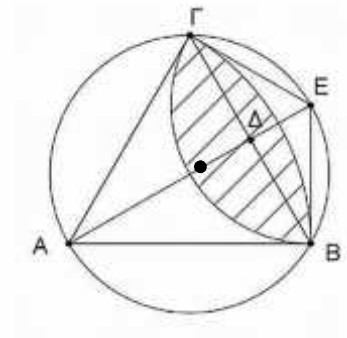
$$\mu \cdot \mu = \frac{\sqrt{3}}{6}$$

$$\mu = \frac{\sqrt{3}}{6}$$



$$O = \frac{3}{2} \quad A = R + \frac{3}{2} = R + \frac{R}{2} = \frac{3R}{2} \quad \mu \quad E = 2R - A = \frac{R}{2}$$

$$\frac{R}{2} = \frac{\sqrt{3}}{6} \Leftrightarrow R = \frac{\sqrt{3}}{3}$$



$$P_3 = 3 \cdot \frac{3}{2} = 3R\sqrt{3} = 3 \cdot \frac{\sqrt{3}}{3} \sqrt{3} = 3$$

$$E = \pi R^2 = \pi \left(\frac{\alpha\sqrt{3}}{3} \right)^2 = \frac{\pi\alpha^2}{3}$$

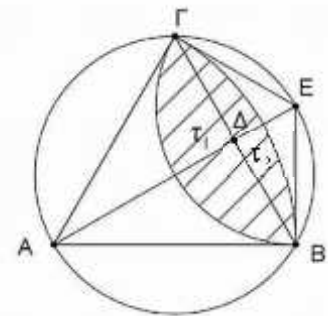
$$\hat{A} = 90^\circ \quad \mu \quad \mu \quad \hat{B} = 30^\circ$$

$$\frac{\mu}{2} = R = \frac{\sqrt{3}}{3} \quad \mu = \frac{\sqrt{3}}{3}$$

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$$= \frac{\mu}{2} + \frac{\mu}{2} + \frac{\mu}{2} = 2 \cdot \frac{\mu}{2} + \frac{\mu}{2} = 2R\sqrt{3} + 2 \cdot \frac{\sqrt{3}}{3} = 2 \cdot \frac{\sqrt{3}}{3} \sqrt{3} + 2 \cdot \frac{\sqrt{3}}{3} = \frac{2(3 + \sqrt{3})}{3}$$

$$(ABE) = 2 \left(\frac{\mu}{2} \right) = 2 \cdot \frac{1}{2} \cdot \mu = R\sqrt{3} \cdot R = R^2\sqrt{3} = \left(\frac{\sqrt{3}}{3} \right)^2 \sqrt{3} = \frac{2\sqrt{3}}{3}$$



$$\mu \quad \mu \quad = \mu_1 + \mu_2 \Leftrightarrow$$

$$= \left(\frac{\mu}{2} \right) - \left(\frac{\mu}{2} \right) + \left(\frac{\mu}{2} \right) - \left(\frac{\mu}{2} \right) \Leftrightarrow$$

$$= \frac{\mu \cdot 120^\circ}{360^\circ} - \frac{1}{2} \cdot \mu \cdot 120^\circ + \frac{\mu \cdot 60^\circ}{360^\circ} - \frac{2\sqrt{3}}{4} \Leftrightarrow$$

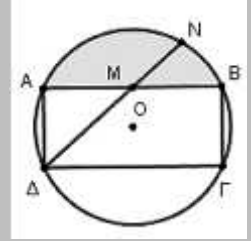
$$= \frac{\left(\frac{\sqrt{3}}{3} \right)^2}{3} - \frac{1}{2} \left(\frac{\sqrt{3}}{3} \right)^2 \cdot \frac{\sqrt{3}}{2} + \frac{(R\sqrt{3})^2}{6} - \frac{(R\sqrt{3})^2 \sqrt{3}}{4} \Leftrightarrow$$

$$E = \frac{2}{9} - \frac{2\sqrt{3}}{12} + \frac{R^2}{2} - \frac{3R^2\sqrt{3}}{4} = \frac{2(4 - 3\sqrt{3})}{36} + \frac{R^2(2 - 3\sqrt{3})}{4} \Leftrightarrow$$

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$$E = \frac{2(4 - 3\sqrt{3})}{36} + \frac{\left(\frac{\sqrt{3}}{3} \right)^2 (2 - 3\sqrt{3})}{4} = \dots = \frac{2(10 - 12\sqrt{3})}{36}$$

25. $(\text{---}, R)$ $\mu\mu$ μ μ $= R$.
 μ , μ :
) μ , $\mu\mu$.
) μ $\mu\mu$ $\mu\mu$.

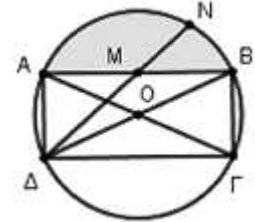


) $AB^2 = R^2 - R^2 = 4R^2 - R^2 = 3R^2 \Leftrightarrow AB = R\sqrt{3}$.

$R^2 = R^2 + \left(\frac{R\sqrt{3}}{2}\right)^2 = R^2 + \frac{3R^2}{4} = \frac{7R^2}{4} \Leftrightarrow R = \frac{R\sqrt{7}}{2}$

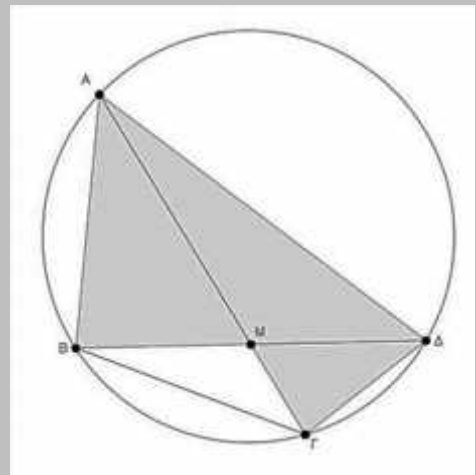
) $\mu = R\sqrt{3}$ $= \frac{1}{3} \cdot 360^\circ = 120^\circ$.

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$= \left(\frac{1}{3} \cdot 360^\circ\right) - \left(\frac{1}{2} \cdot 360^\circ\right) = \frac{R^2 \cdot 120^\circ}{360^\circ} - \frac{1}{2} R^2 = \frac{R^2}{3} - \frac{1}{2} R^2 = \frac{2R^2 - 3R^2}{6} = -\frac{R^2}{6}$

26. $\mu\mu$ μ μ μ $AM = \frac{4}{5} \mu$.
) $(\text{---}) = 4(\text{---})$
) $\hat{\quad} = 30^\circ$:
 i. $(\text{---}) = \frac{2}{32} \mu$.
 ii. $(\text{---}) = \frac{9}{32} \mu$.



) $\hat{\quad} + \hat{\quad} = 180^\circ$:

$\frac{(\text{---})}{(\text{---})} = \frac{4}{1} = 4 \Leftrightarrow (\text{---}) = 4(\text{---})$

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i. $(\text{---}) = \frac{1}{2} \mu \cdot \mu 30^\circ = \frac{1}{2} \cdot \frac{1}{5} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{40} \cdot \frac{1}{4} = \frac{1}{160} \mu$

ii. $(\text{---}) = (\text{---}) + (\text{---}) + (\text{---}) = 2(\text{---}) + (\text{---}) = 8(\text{---}) + (\text{---}) = 9(\text{---}) = \frac{9}{32} \mu$

)
$$2 = \left(\frac{4\sqrt{3}}{3} \right)^2 - 2^2 = \frac{4}{3} \Leftrightarrow \frac{2\sqrt{3}}{3}$$



)
$$\frac{2\sqrt{3}}{3} = \frac{1}{2} \Rightarrow \hat{\alpha} = 30^\circ$$

$$\hat{\beta} = \hat{\gamma} = 30^\circ, \hat{\delta} = 180^\circ - 2 \cdot 30^\circ = 120^\circ$$

)
$$1 = \left(\frac{4\sqrt{3}}{3} \right)^2 - \frac{4^2 \cdot 30^\circ}{360^\circ \cdot 12} - \frac{1}{2} \cdot \mu 120^\circ = \frac{4}{3} - \frac{1}{2} \left(\frac{4\sqrt{3}}{3} \right)^2 \cdot \frac{\sqrt{3}}{2} = \dots = \frac{4}{3} (-\sqrt{3})$$

)
$$2 = \left(\frac{4\sqrt{3}}{3} \right)^2 - \frac{4^2 \cdot 60^\circ}{360^\circ \cdot 6} - \frac{4}{3} (-\sqrt{3}) = \frac{4(3\sqrt{3} -)}{9}$$